MULTIPLE CHOICE QUESTIONS

1. A 4.9kg mass is initially at rest on a surface with $\mu_s = 0.55$ and $\mu_k = 0.45$. A 30N force is then applied to the mass. What is the following magnitude, and type, of friction acting on the mass?

   (a) 27N, static

   (b) 30N, static

   (c) 22N, kinetic

   (d) 30N, kinetic

$\boxed{49} \rightarrow 30N > f_{s,max} \Rightarrow \boxed{\text{FRICTION IS KINETIC}}$ $\quad f_k = \mu_k N = (0.45)(49) = \boxed{22N}$

$N = (4.9)(10) = 49N \Rightarrow f_{s,max} = \mu_s N = (0.55)(49) = 27N$

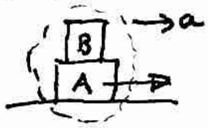
2. Box B is placed on top of box A. If box A is pushed to the right such that box boxes accelerate together, is there a friction on box B?

   (a) There is a kinetic friction, to the right

   (b) There is a static friction, to the right

   (c) There is a static friction, to the left

   (d) There is no friction on box B because it isn't sliding



MUST BE A STATIC FRICTION ON B TO THE RIGHT TO PRODUCE THE ACCELERATION.

3. Under what conditions is the energy of an object conserved?

   (a) If only gravity and the normal force act on an object

   (b) If only conservative forces act on an object

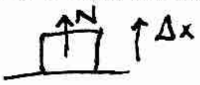
   (c) If the work due to non-conservative forces acting on the object is zero

   (d) Energy is a conserved quantity, so it's always conserved

4. An elevator lifts a box. During this lift, the energy of the box is conserved..

   (a) True

   (b) False



IF $\Delta\vec{x}$ IS UP, THEN N DOES WORK, SO ENERGY IS NOT CONSERVED.

5. A 500g ball is held against a horizontal, 100 N/m spring compressed by 12cm. What speed will be the ball be fired at when the spring is released?

   (a) 0.72 m/s

   (b) 1.70 m/s

   (c) 2.50 m/s

   (d) 3.14 m/s

$U_i = K_f = \frac{1}{2}mv^2$

$U_i = \frac{1}{2}kx^2 = \frac{1}{2}(100)(0.12)^2 = 0.72\,J$

$\Rightarrow v = \sqrt{\frac{2U_i}{m}} = \sqrt{\frac{2(0.72)}{(0.5)}} = \boxed{1.70\,m/s}$

6. A mass slides down an incline under the influence of friction, at a constant velocity. The total work done on this mass should be:

   (a) Positive

   (b) Negative

   (c) Zero

   (d) Impossible to tell without numbers

$W_{tot} = \Delta K = 0$

B/C OF CONSTANT V!

7. A 2.5kg ball bounces off a wall horizontally. If it hits the wall at 12 m/s, and leaves the wall at 7 m/s, how much impulse did the wall deliver to the ball?

   (a) 5 Ns
   (b) 12.5 Ns
   (c) 19 Ns
   (d) 47.5 Ns

IMPULSE/MOMENTUM ARE VECTORS!!

BEFORE: O→| -12 m/s

AFTER: ←O| $+7$ m/s

$\Delta p = mv_f - mv_i = m(v_f - v_i)$

$= (2.5)[7-(-12)] = \boxed{47.5\ Ns}$

8. Which of the following is an important consequence of Newton's third law?

   (a) An object's momentum will only change if a force acts upon it
   (b) The net internal force on any system is always zero
   (c) The net external force on any system is always zero
   (d) Momentum is always conserved

9. When is the momentum of a system conserved?

   (a) Momentum is a conserved quantity, so it's always conserved
   (b) Only during collisions
   (c) Only if the net internal force on the system is zero
   (d) Only if the net external force on the system is zero

10. Initially, a 55g piece of clay rolls at 25 cm/s to the right, while an 85g piece of clay rolls at 20 cm/s to the left. During the collision, the two pieces of clay stick together. After the collision, in what direction does the lump of clay move?

   (a) To the left
   (b) To the right
   (c) It's stopped by the collision
   (d) None of the above

DIRECTION OF FINAL MOMENTUM MUST EQUAL DIRECTION OF INITIAL

$P_i = P_{1i} + P_{2i} = m_1 v_{1i} + m_2 v_{2i}$

→ + (RIGHT IS POSITIVE)

1: (55g) → 25 cm/s

2: ← (85g) -20 cm/s

SI UNITS

$P_i = (0.055)(0.25) + (0.085)(-0.20)$

$= -0.00325$ Ns

$\boxed{\text{TO THE LEFT!}}$

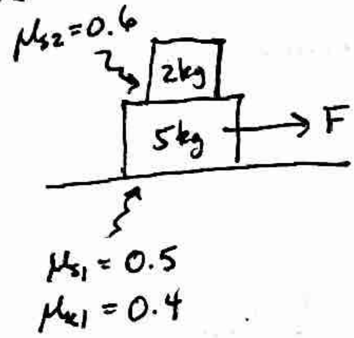
FREE-RESPONSE PROBLEMS

1. A 2kg box is placed on top of a 5kg box. Between the 5kg box and the ground are the coefficients $\mu_{s1} = 0.5$ and $\mu_{k1} = 0.4$, and between the 2kg box and the 5kg box is the coefficient $\mu_{s2} = 0.6$.

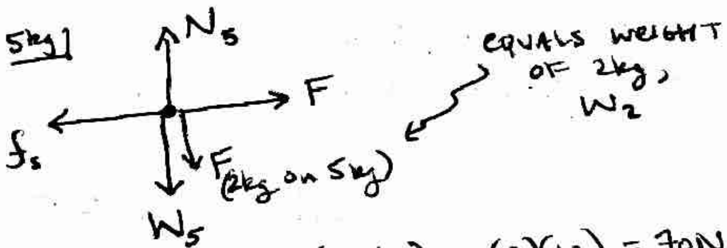
   a) If the 5kg box is pushed by a horizontal, 20N force, is there any friction acting on the 2kg box? If so, what type and what magnitude?

   b) If the 5kg box is pushed by a horizontal, 50N force, is there any friction acting on the 2kg box? If so, what type and what magnitude?

   c) What is the maximum horizontal force that can be applied on the 5kg box before the 2kg slips off of the 5kg box?



a) Does F = 20N overcome static friction?



$\Rightarrow N_5 = W_5 + F_{2\,on\,5} = (5)(10) + (2)(10) = 70N$

$\Rightarrow f_{s,max} = \mu_{s1} N_5 = (0.5)(70) = 35N$

20N is too small, so $\boxed{\text{No friction on 2kg}}$

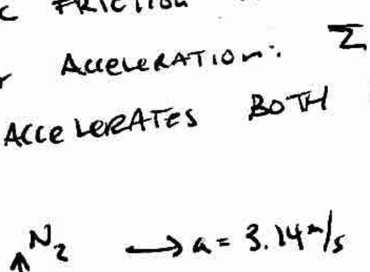
b) A 50N force is ~~...~~ larger than $f_{s,max}$, so boxes both accelerate.

2kg needs static friction to accelerate with 5kg.

Net force causing acceleration: $\Sigma F = F - f_k = 50 - \mu_{k1} N_5 = 22N$

The 22N force accelerates both boxes, so $a = \frac{\Sigma F}{m_{tot}} = \frac{22}{5+2} = 3.14\, m/s^2$

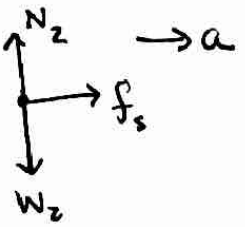
For 2kg,



$\Rightarrow f_s = m_2 a = (2)(3.14) = \boxed{6.28N}$

c) MAX F $\Rightarrow$ MAX a $\Rightarrow$ $f_{s,MAX}$ on 2kg

2kg]



$$N_2 = W_2 = (2)(10) = 20N$$

$$\Rightarrow f_{s,MAX} = \mu_{s2} N_2 = (0.6)(20) = 12N$$

So, IF $f_{s,MAX}$ on 2kg is 12N, THEN MAX ACCELERATION is

$$a = \frac{f_{s,MAX}}{2kg} = \frac{12}{2} = 6\ m/s^2$$

WHAT NET FORCE WOULD PRODUCE THIS? REMEMBER, THIS ACCELERATES BOTH BOXES:

$$\Sigma F = m_{tot}\, a = (7)(6) = 42N$$

We Know $\Sigma F = F - f_k$ AND $f_k = \mu_{k1} N_5 = (0.4)(70) = 28N$

So,

$$F = \Sigma F + f_k = 42 + 28$$

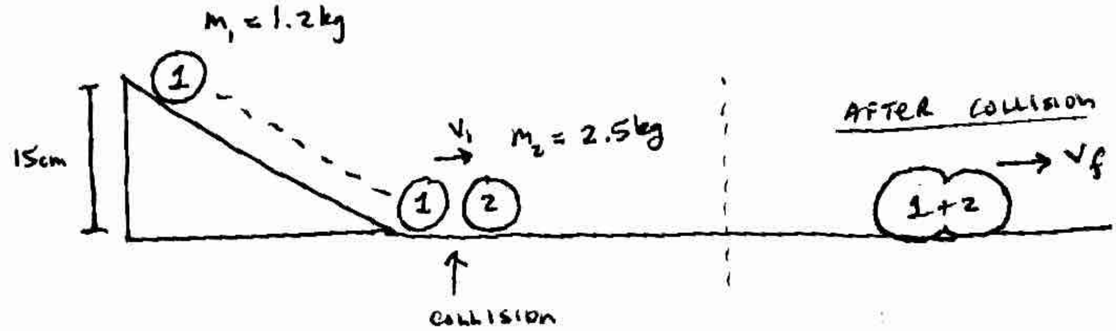
$$\Rightarrow \boxed{F = 70N}$$

2. A 1.2kg ball of clay rolls down a ramp from a height of 15cm. At the bottom of the ramp, it collides with a second ball of clay, of mass 2.5kg, causing them to stick together.

a) At the bottom of the ramp, what is the speed of the 1.2kg ball of clay?

b) After they collide, what is the speed of the combined lump of clay?

c) Is energy conserved throughout this entire process? If not, how much energy is lost from start to finish?



a) AS 1 ROLLS DOWN INCLINE, ENERGY IS CONSERVED, SO:

$$\cancel{K_i}^{\,0} + U_i = K_f + \cancel{U_f}^{\,0} \qquad (K_i = 0 \text{ \& SET } y=0 \text{ @ BOTTOM OF RAMP})$$

$$U_i = m_1 g h = (1.2)(10)(0.15) = 1.8\text{J}$$

$$\Rightarrow K_f = 1.8\text{J} = \tfrac{1}{2} m_1 v_1^2 \Rightarrow v_1 = \sqrt{\frac{2K_f}{m_1}} = \sqrt{\frac{2(1.8)}{(1.2)}} = \boxed{1.73 \text{ m/s}}$$

b) COLLISION IS PERFECTLY INELASTIC:

$$m_1 v_{1i} + m_2 \cancel{v_{2i}}^{\,0} = (m_1 + m_2) v_f \qquad (m_2 \text{ @ REST INITIALLY})$$

$$\Rightarrow v_f = \frac{m_1 v_{1i}}{m_1 + m_2} = \frac{(1.2)(1.73)}{1.2 + 2.5} = \boxed{0.56 \text{ m/s}}$$

c) COLLISION IS <u>INELASTIC</u>, SO ENERGY IS <u>LOST</u>. INITIAL ENERGY IS ALL POTENTIAL CONTAINED BY m_1, SO $E_i = U_i = 1.8$J. FINAL ENERGY IS ALL KINETIC CONTAINED BY $m_1 + m_2$, SO:

$$E_f = K_f = \tfrac{1}{2}(m_1 + m_2) v_f^2 = \tfrac{1}{2}(1.2 + 2.5)(0.56)^2 = 0.58\text{J}$$

SO, $1.8 - 0.58 = \boxed{1.22\text{J IS LOST}}$

3. A horizontal spring, of force constant 200 N/m, is placed in front of a ramp, such that a 175g plastic ball can be propelled by the spring and roll up the ramp. Before the ball is fired, the spring is compressed by 10cm.

a) Ignoring any friction or air resistance, how fast is the plastic ball fired from the spring?

b) If friction does -0.15J of work while the ball rises up the ramp, what is the maximum height the ball will roll up the ramp to?

c) If friction does the same amount of work on the way down, and the ball hits the spring again at the bottom of the ramp, how far will the ball compress the spring when it returns?

a)

NO AIR RESISTANCE OR FRICTION $\Rightarrow$ ENERGY IS CONSERVED

$$U_i = \frac{1}{2}kx^2 = \frac{1}{2}(200)(0.1)^2 = 1\,J$$

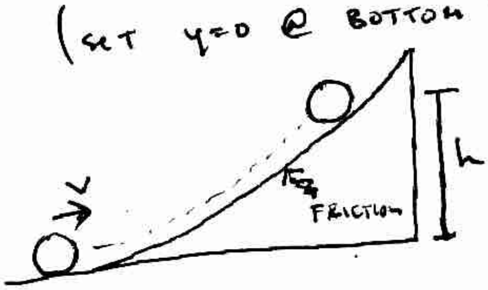
$$\Rightarrow K_f = 1\,J = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(1)}{(0.175)}} = \boxed{3.38\,m/s}$$

b) NOW, AS IT GOES UP THE RAMP, ENERGY IS NO LONGER CONSERVED B/C FRICTION DOES WORK $W_f = -0.15\,J$.

(SET $y=0$ @ BOTTOM OF RAMP)

$$K_i + \cancel{U_i}^{\,0} + W_{nc} = \cancel{K_f}^{\,0} + U_f$$

$$\Rightarrow K_i + W_{nc} = U_f$$



WE KNOW $K_i = 1\,J$ FROM (a)

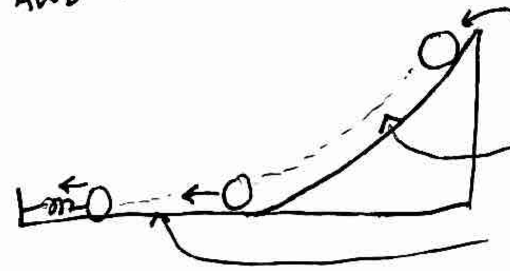
AND WE WERE TOLD $W_{nc} = -0.15\,J$,

SO

$$U_f = 1 - 0.15 = 0.85\,J$$

AND AT A HEIGHT $y=h$, $U = mgh \Rightarrow h = \frac{U}{mg} = \frac{0.85}{(.175)(10)} = \boxed{0.49\,m}$

c)



STARTS WITH 0.85 J OF U

LOSES 0.15 J DUE TO FRICTION

ENERGY IS CONSERVED AS SPRING COMPRESSES.

SO, THE BALL HITS THE SPRING WITH $0.85 - 0.15 = 0.7\,J$ OF KINETIC ENERGY. SO AFTER COMPRESSING, THE SPRING WILL HAVE $U = 0.7\,J$, AND:

$$U = \frac{1}{2}kx^2 \Rightarrow x = \sqrt{\frac{2U}{k}} = \sqrt{\frac{2(0.7)}{(200)}} = \boxed{0.084\,m = 8.4\,cm}$$

4. A 1200kg car, moving at 25 m/s to the right, collides with a 2500kg truck, moving at 15 m/s to the left.

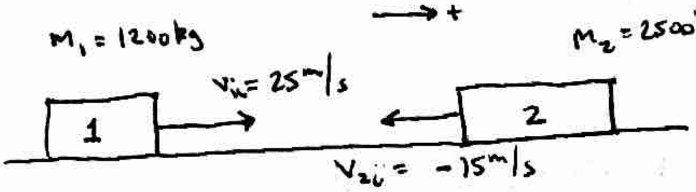
   a) If the car recoils at 15 m/s to the left, what is the final speed and direction of the truck?

   b) If the collision between the car and truck is perfectly inelastic, what is the final speed and direction of each object?

   c) If the collision is elastic, what is the final speed and direction of each object?

   d) What is the maximum amount of heat that can be released during a collision between these two objects?

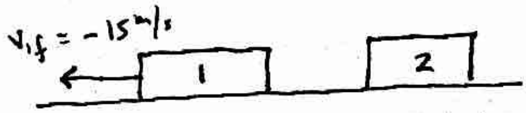
BEFORE |



DIFFERENT "AFTER" PICTURE FOR EACH PART.

a)

AFTER |



INELASTIC COLLISION ⇒ ONLY MOMENTUM CONSERVATION:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\Rightarrow (1200)(25) + (2500)(-15) = (1200)(-15) + 2500\, v_{2f}$$

$$\Rightarrow -7500 = -18{,}000 + 2500\, v_{2f}$$

$$\Rightarrow v_{2f} = \frac{10{,}500}{2500} = \boxed{+4.2\text{ m/s}}$$ (TO THE RIGHT)

b) PERFECTLY INELASTIC ⇒ STICK TOGETHER, & MOMENTUM EQUATION IS:

$$\underbrace{m_1 v_{1i} + m_2 v_{2i}}_{\text{CALCULATED TO BE } -7500 \text{ IN (a)}} = (m_1 + m_2) v_f$$

$$\Rightarrow -7500 = (1200 + 2500) v_f$$

$$\Rightarrow v_f = \boxed{-2.03\text{ m/s}}$$ (TO THE LEFT)

c) ELASTIC COLLISION ⇒ USE ~~[illegible]~~ SPECIAL ELASTIC EQUATION:

$$v_{1i} + v_{1f} = v_{2i} + v_{2f}$$

$$\Rightarrow 25 + v_{1f} = -15 + v_{2f}$$

$$\Rightarrow v_{2f} = v_{1f} + 40$$

NOW, PLUG INTO MOMENTUM CONSERVATION EQUATION:

$$\underbrace{m_1 v_{1i} + m_2 v_{2i}}_{-7500 \text{ FROM (a)}} = m_1 v_{1f} + m_2 v_{2f}$$

$$\Rightarrow -7500 = 1200 v_{1f} + 2500(v_{1f} + 40)$$

$$= 1200 v_{1f} + 2500 v_{1f} + 100{,}000$$

$$= 3700 v_{1f} + 100{,}000$$

$$\Rightarrow v_{1f} = \frac{-107{,}500}{3700} = \boxed{-29 \text{ m/s}} \text{ (TO THE LEFT)}$$

NOW, PLUG THIS BACK INTO OUR SUBSTITUTION EQUATION FOR v_{2f}:

$$v_{2f} = v_{1f} + 40 = -29 + 40 = \boxed{11 \text{ m/s}} \text{ (TO THE RIGHT)}$$

d) MAX HEAT ⇒ MAX ENERGY LOST ⇒ PERFECTLY INELASTIC COLLISION

INITIAL: $K_i = \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2}(1200)(25)^2 + \frac{1}{2}(\cancel{[illegible]}^{2500})(15)^2 = 656{,}250 \text{ J}$

FINAL (FROM PART b): $K_f = \frac{1}{2}(m_1 + m_2) v_f^2 = \frac{1}{2}(1200 + 2500)(2.03)^2 = 7{,}624 \text{ J}$

SO, ENERGY LOST, OR HEAT RELEASED, IS:

$$656{,}250 - 7624 = \boxed{648{,}626 \text{ J}}$$