PHY2054 General Physics II

Section 611820

Prof. Douglas H. Laurence

Exam 1 (Chapters 19 – 21)

February 25, 2018

Name: Solutions

Instructions:

This exam is composed of **10 multiple choice questions** and **4 free-response problems**. To receive a perfect score (100) on this exam, 3 of the 4 free-response problems must be completed. The fourth free-response problem **may not be answered for extra credit.** Each multiple choice question is worth 2.5 points, for a total of 25 points, and each free-response problem is worth 25 points, for a total of 75 points. This means that your exam will be scored out of 100 total points, which will be presented in the rubric below. **Please do not write in the rubric below; it is for grading purposes only.**

Only scientific calculators are allowed – do not use any graphing or programmable calculators.

For multiple choice questions, no work must be shown to justify your answer and no partial credit will be given for any work. However, for the free response questions, **work must be shown to justify your answers.** The clearer the logic and presentation of your work, the easier it will be for the instructor to follow your logic and assign partial credit accordingly.

The exam begins on the next page. **The formula sheet is attached to the end of the exam.**

Exam Grade:

Multiple Choice	
Problem 1	
Problem 2	
Problem 3	
Problem 4	
Total	

MULTIPLE CHOICE QUESTIONS

1. Most materials in nature are found to be:

   (a) Positively charged

   (b) Negatively charged

   (c) Neutral

   (d) It's random

2. A charge $Q = 1.6$ nC is composed of: (nano $= 10^{-9}$)

   (a) 10^{10} excess protons

   (b) 10^{-10} excess protons

   (c) 10^{10} excess electrons

   (d) 10^{-10} excess electrons

   $Q = Ne$ (FOR MAGNITUDE)

   $\Rightarrow N = Q/e = (1.6\times10^{-9})/(1.6\times10^{-19}) = 10^{10}$

   SINCE $Q = (+)1.6$ nC, THESE ARE EXCESS PROTONS

3. Consider a charge q_A producing an electric field. A second charge q_B feels the electric field with a magnitude E. If the distance between q_A and q_B is halved, and the charge q_B is doubled, what is the new value of the electric field felt by q_B? E FELT BY q_B IS PRODUCED BY q_A:

   (a) $E/4$

   (b) $E/2$

   (c) $2E$

   (d) $4E$

   $E_A = k\frac{q_A}{r^2}$; IF DISTANCE IS HALVED, $r \to r/2$; IF q_B IS DOUBLED, E_A DOESN'T CHANGE (IT DEPENDS ON q_A)

   $\Rightarrow E_A' = k\frac{q_A}{(r/2)^2} = \frac{1}{4}\left(k\frac{q_A}{r^2}\right) = \frac{1}{4}E_A$

4. Three possible surfaces enclose the same charge q. Surface S_1 is a sphere of radius 2cm with q at its center; surface S_2 is a sphere of radius 2cm with q 1cm off-center; surface S_3 is a cube of side-length 2cm with q at its center. Which surface has the greatest flux passing through it?

   (a) S_1

   (b) S_2

   (c) S_3

   (d) They all have the same flux through their surfaces

   GAUSS' LAW: FLUX THROUGH SURFACE DEPENDS ONLY ON CHARGE ENCLOSED, NOT ON SHAPE OF SURFACE OR POSITION OF CHARGE W/IN.

5. Electrons will always move towards: EVERYTHING MOVES TOWARDS LOW POTENTIAL ENERGY

   (a) High potential energy and high potential

   (b) High potential energy but low potential

   (c) Low potential energy but high potential

   (d) Low potential energy and low potential

   $\Delta U = q\Delta\phi$
   $(-)\quad (-)(+)$, SO $\Delta\phi$ MUST BE (+) (FOR ELECTRONS)
   $\to \phi_f > \phi_i$ HIGHER ϕ

6. Charges are separated such that all positive charges accumulate at some point A and all negative charges accumulate at some point B. Which of the following statements is true?

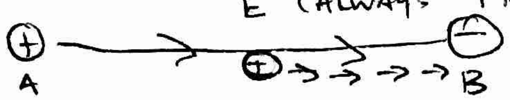
   (a) The electric field points from A to B, with A being the point of low potential

   (b) The electric field points from A to B, with B being the point of low potential

   (c) The electric field points from B to A, with A being the point of low potential

   (d) The electric field points from B to A, with B being the point of low potential

   $\vec{E}$ (ALWAYS FROM $\oplus \to \ominus$) FROM A TO B



IF I PLACE A $\oplus$ BETWEEN A & B, IT WILL MOVE TO B (THE $\ominus$), SO B IS AT LOW POTENTIAL

MICRO = 10^{-6}

7. A 15 μF capacitor is connected to a battery of 5V. What is the maximum charge on the capacitor when connected to this battery? AT MAX CHARGE, $V_C = V_{BATTERY} = 5V$

   (a) 0.33 μC
   (b) 3 μC
   (c) 75 μC
   (d) 75 C

$Q = CV = (15 \times 10^{-6})(5) = 75 \times 10^{-6}\,C$ ↳ 10^{-6} = MICRO

$= \boxed{75\,\mu C}$

8. How much energy would a battery have to output to charge a 5F capacitor to 10C?

   (a) 2J
   (b) 10J
   (c) 25J
   (d) 50J

ENERGY STORED BY CAPACITOR = ENERGY SPENT BY BATTERY

$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{(10)^2}{(5)} = \boxed{10\,J}$

9. Resistance is a quantity that should depend upon:

   (a) Geometry of the resistor alone
   (b) Geometry and the material of the resistor
   (c) Geometry, the material, and the orientation of the resistor
   (d) None of the above

$R = \rho\left(\frac{L}{A}\right)$ ← LENGTH & AREA DEPEND UPON GEOMETRY AND ORIENTATION.

↳ RESISTIVITY DEPENDS ON MATERIAL

10. A capacitor with some area A and some plate separation d has a capacitance of C. When the capacitor is connected to a battery of voltage V and fully charged, there is an electric field of E between its plates. If the capacitor's plates are brought together to a distance of $d/2$, while the capacitor is still connected to the battery, the new electric field between the capacitor plates will be:

   (a) $E/4$
   (b) $E/2$
   (c) $2E$
   (d) $4E$

IF CAPACITOR IS CONNECTED TO A BATTERY OF VOLTAGE V, THE CAPACITOR'S VOLTAGE IS ALWAYS V (B/C THEY'RE IN PARALLEL).

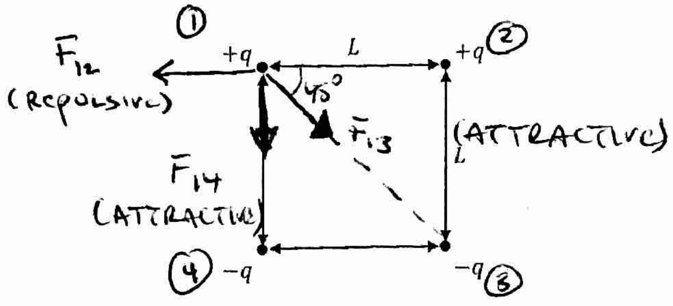
SO WHEN PLATES CHANGE DISTANCE V STAYS THE SAME. THE ORIGINAL ELECTRIC FIELD IS:

$E = \frac{V}{d}$

THE NEW ELECTRIC FIELD IS:

$E' = \frac{V}{(d/2)} = 2\left(\frac{V}{d}\right) = \boxed{2E}$

FREE-RESPONSE PROBLEMS



1. Imagine four charges arranged in the corners of an $L \times L$ square, as shown in the figure above. Consider the case of $q = e$ and $L = 10\text{cm}$.

   (a) What is the electric force on the upper-left charge?

   (b) What is the electric force on the lower-left charge? *Hint: you can use symmetry arguments to answer this question based on how it relates to part (a).*

   (c) Imagine placing a fifth charge, $+q$, in the figure above. Where would it need to be placed so that the electric force on the upper-left charge is zero?

(a) ALL ELECTRIC FORCES ARE GIVEN BY COULOMB'S LAW:

$$F_{12} = k\frac{q_1 q_2}{r_{12}^2}$$

q_1, q_2 | +q ------ L ------ +q

$$F_{12} = k\frac{q_1 q_2}{r_{12}^2} = k\frac{q^2}{L^2}$$

MAGNITUDES ARE THE SAME.

q_1, q_4 | +q, L, −q

$$F_{14} = k\frac{q_1 q_4}{r_{14}^2} = k\frac{q^2}{L^2}$$

$\frac{1}{2}$ OF F_{12} & F_{14}

q_1, q_3 | +q, L, 45°, L, −q

$$\sqrt{L^2 + L^2} = \sqrt{2}\,L$$

$$F_{13} = k\frac{q_1 q_3}{r_{13}^2} = k\frac{q^2}{(\sqrt{2}L)^2} = \frac{1}{2}k\frac{q^2}{L^2}$$

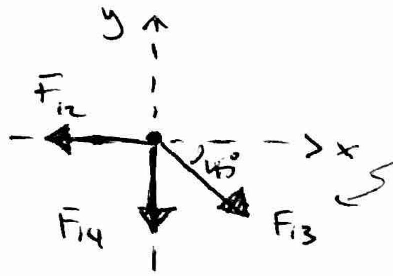
Let's compute $F = k\frac{q^2}{L^2} = (8.99\times10^9)\frac{(1.6\times10^{-19})^2}{(0.1)^2}$

CONTINUED ON BACK

$\approx 2.3\times10^{-26}\,N$

(a) CONTINUED

DRAWING THE FORCES ACTING ON UPPER-LEFT CHARGE:



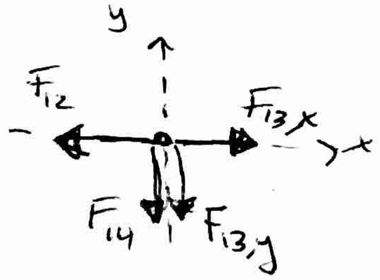
WE NEED TO BREAK F_{13} UP INTO ITS x & y COMPONENTS TO FIND THE TOTAL FORCE:

$$F_{13,x} = F_{13}\cos 45 = \frac{\sqrt{2}}{2} F_{13}$$

$$F_{13,y} = F_{13}\sin 45 = \frac{\sqrt{2}}{2} F_{13}$$

THE SAME:

$$\frac{\sqrt{2}}{2} F_{13} = \frac{\sqrt{2}}{2}\left(\frac{1}{2} k \frac{q^2}{L^2}\right) = \frac{\sqrt{2}}{4} k \frac{q^2}{L^2}$$



SO, THE NET FORCE, ΣF, HAS COMPONENTS:

$$\Sigma F_x = F_{12} - F_{13,x} = k\frac{q^2}{L^2} - \frac{\sqrt{2}}{4} k \frac{q^2}{L^2} = \left(1 - \frac{\sqrt{2}}{4}\right) k \frac{q^2}{L^2}$$

$$\Sigma F_y = F_{14} + F_{13,y} = k\frac{q^2}{L^2} + \frac{\sqrt{2}}{4} k \frac{q^2}{L^2} = \left(1 + \frac{\sqrt{2}}{4}\right) k \frac{q^2}{L^2}$$

(FROM PREV. PAGE)

USING $F = k\frac{q^2}{L^2} = 2.3 \times 10^{-26} N$,

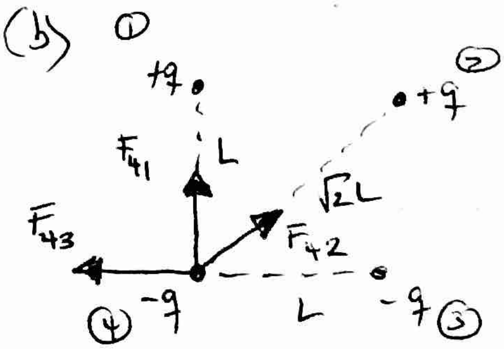
$$\Sigma F_x = \left(1 - \frac{\sqrt{2}}{4}\right)(2.3 \times 10^{-26}) = 1.49 \times 10^{-26} N$$

$$\Sigma F_y = \left(1 + \frac{\sqrt{2}}{4}\right)(2.3 \times 10^{-26}) = 3.11 \times 10^{-26} N$$

$$\Rightarrow \Sigma F = \sqrt{\Sigma F_x^2 + \Sigma F_y^2} = \sqrt{(1.49 \times 10^{-26})^2 + (3.11 \times 10^{-26})^2}$$

$$= \boxed{3.45 \times 10^{-26} N}$$

(b)

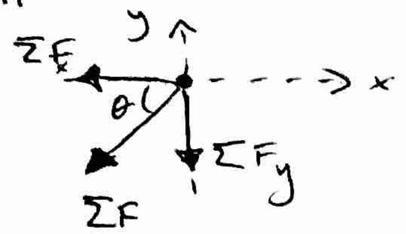


THIS IS IDENTICAL TO PART (a), JUST W/ THE FORCES POINTING IN DIFFERENT DIRECTIONS. SO,

$$\boxed{\Sigma F = 3.45 \times 10^{-26} N}$$

CONTINUE ON NEXT PAGE

(c) Let's look back at the net force on the upper left charge:



where

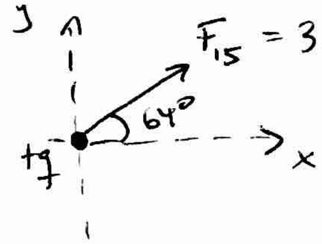
$$\Sigma F_x = 1.49 \times 10^{-26}\,N$$

$$\Sigma F_y = 3.11 \times 10^{-26}\,N$$

The angle of the net force, θ, is:

$$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = \tan^{-1}\left(\frac{3.11\times10^{-26}}{1.49\times10^{-26}}\right) = 64^\circ$$

To cancel out ΣF, we need a fifth charge to produce the exact same force in the opposite direction:

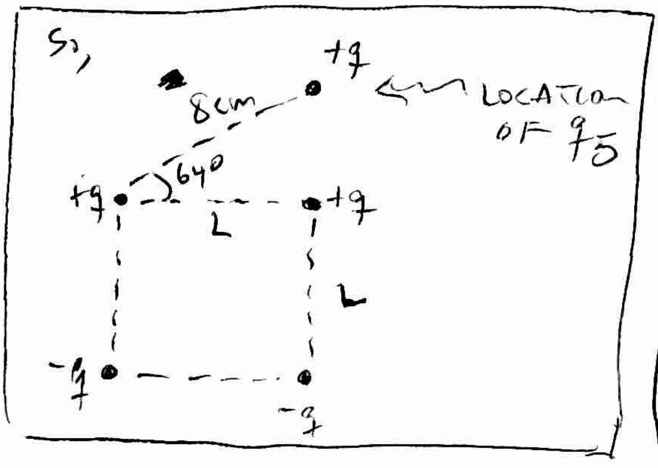


$F_{15} = 3.45\times10^{-26}\,N$ (from part a)

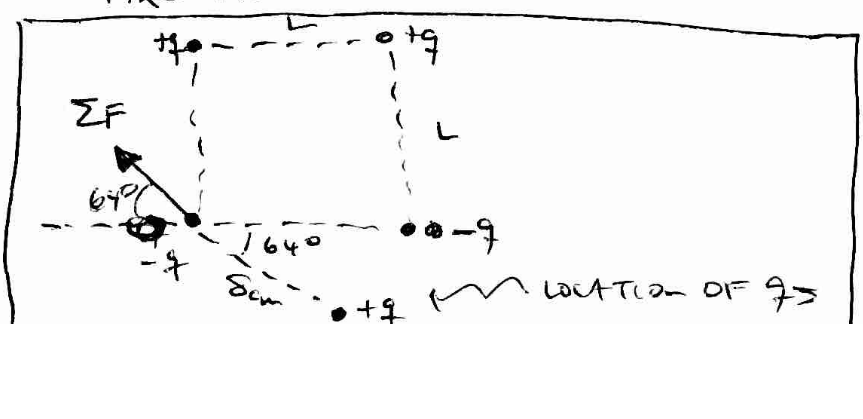
We need to know how far to place q_5 ($=+q$, as stated)

$$F = k\frac{q_1 q_5}{r_{15}^2}$$

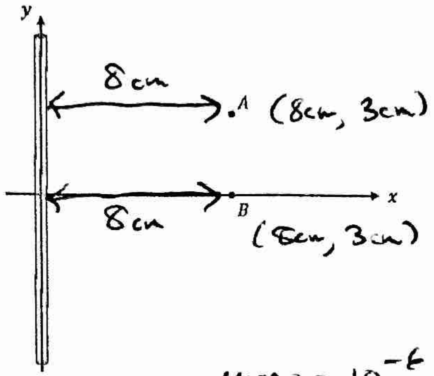
$$\Rightarrow r_{15} = \sqrt{\frac{k q_1 q_5}{F}} = \sqrt{\frac{kq^2}{F}} = \sqrt{\frac{(8.99\times10^9)(1.6\times10^{-19})^2}{(3.45\times10^{-26})}} = 0.08\,m = 8\,cm$$



d) On lower-left charge, everything is the same, just in a different direction:



FOR WIRE, ALL THAT MATTERS IS THE HORIZONTAL DISTANCE:



MICRO = 10^{-6}

2. A very long wire has a total charge of 15 μC and a length of 1.2m. The center of the wire is placed at the origin of a coordinate system, and the wire runs along the y-axis, as shown in the figure above.

(a) What is the electric field at the point A in the above figure, located at (8cm, 3cm)?

(b) What is the electric field at the point B in the above figure, located at (8cm, 0)?

NANO = 10^{-9}

(c) If a charge $q = 5$ nC was placed at point A, what would the electric force on q be?

(d) If the same charge q was placed at point B, instead, what would the electric force on q be?

(a) $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$ FOR A VERY LONG (INFINITE) WIRE.

$\lambda = \dfrac{Q}{L} = \dfrac{15\mu C}{1.2m} = 12.5 \times 10^{-6} \dfrac{C}{m}$

$\Rightarrow E_A = \dfrac{(12.5 \times 10^{-6})}{2\pi(8.85\times10^{-12})(0.08)} = \boxed{2.81 \times 10^{6} \dfrac{N}{C}}$

8cm = 0.08m

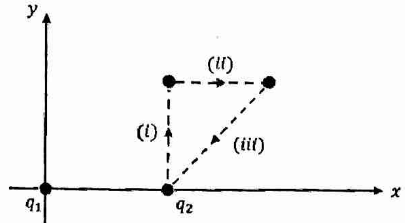
(b) POINT B IS LOCATED AT THE SAME HORIZONTAL DISTANCE AS A, SO $\boxed{E_B = 2.81 \times 10^{6} \dfrac{N}{C}}$

(c) FORCE IS GIVEN BY $F = qE$

$\Rightarrow F_A = qE_A = (5\times10^{-9})(2.81\times10^{6}) = \boxed{0.014N}$

(d) SINCE $E_B = E_A$, THE FORCES ARE THE SAME AS WELL:

$\boxed{F_B = 0.014N}$



← nano = 10^{-9} →

3. A charge $q_1 = -10$ nC is fixed at the origin. A second charge $q_2 = 25$ nC is moved along the following path, as shown in the figure above: (i) q_2 is moved from (10cm, 0) to (10cm, 5cm); (ii) q_2 is then moved from (10cm, 5cm) to (15cm, 5cm); (iii) q_2 is then moved back to its starting point.

(a) How much work does the electric force do on q_2 along path (i)?

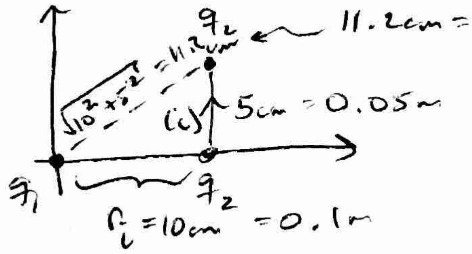
(b) How much work does the electric force do on q_2 along path (ii)?

(c) How much work does the electric force do on q_2 along path (iii)?

(d) What is the work done along the entire path? What does this answer signify about the electric force? *Hint: the total path taken by the charge is a closed loop.*

FOR ALL PATHS, $W = -\Delta U = U_i - U_f$, & U depends only on position, r

(a)



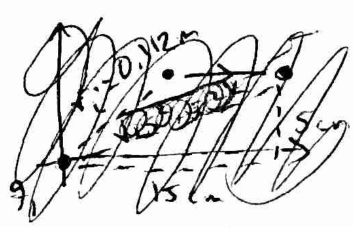
$$W = U_i - U_f = k\frac{q_1 q_2}{r_i} - k\frac{q_1 q_2}{r_f} = kq_1q_2\left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

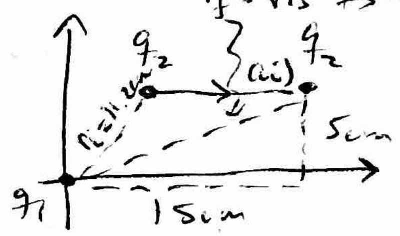
ALL CALCULATIONS ARE GOING TO DEPEND ON:

$$kq_1q_2 = (8.99\times10^9)(-10\times10^{-9})(25\times10^{-9}) = -2.25\times10^{-6}$$ (in SI units)

$$W_{(i)} = kq_1q_2\left(\frac{1}{r_i} - \frac{1}{r_f}\right) = (-2.25\times10^{-6})\left(\frac{1}{0.1} - \frac{1}{0.112}\right) = \boxed{-2.41\times10^{-6}\text{ J}}$$

(b)

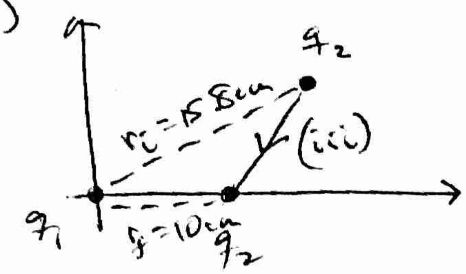




$r_f = \sqrt{15^2+5^2} = 15.8$cm $= 0.158$m

$$W_{(ii)} = kq_1q_2\left(\frac{1}{r_i} - \frac{1}{r_f}\right) = (-2.25\times10^{-6})\left(\frac{1}{.112} - \frac{1}{.158}\right) = \boxed{-5.85\times10^{-6}\text{ J}}$$

(c)



$$W_{(iii)} = kq_1q_2\left(\frac{1}{r_i} - \frac{1}{r_f}\right) = (-2.25\times10^{-6})\left(\frac{1}{.158} - \frac{1}{.1}\right)$$

$$= +\boxed{8.26\times10^{-6}\ J}$$

(d)

$$W_{tot} = W_{(i)} + W_{(ii)} + W_{(iii)} = -2.41\times10^{-6} - 5.85\times10^{-6} + 8.26\times10^{-6}$$

$$= \boxed{0}$$

~~IF THE WORK DONE BY A FORCE AROUND ANY CLOSED LOOP~~

IF A FORCE IS CONSERVATIVE (MEANING IT CONSERVES ENERGY), THEN THE WORK DONE BY THAT FORCE AROUND ANY CLOSED LOOP IS ZERO. THE ELECTRIC FORCE IS, INDEED, CONSERVATIVE, SO WE SHOULD EXPECT THIS ANSWER.

NOTE: POTENTIAL ENERGIES ONLY EXIST FOR CONSERVATIVE FORCES, SO BY USING $W = -\Delta U$, WE ACTUALLY ASSUMED THE FORCE WAS CONSERVATIVE TO SHOW THAT $W_{LOOP} = 0$. THIS ARGUMENT IS TECHNICALLY CIRCULAR, SINCE WE HAD TO ASSUME THE FORCE WAS CONSERVATIVE IN ORDER TO SHOW THAT IT WAS CONSERVATIVE.

4. A cylindrical resistor, made of an unknown material, has a radius of 1mm and a length of 5cm. If, when connected to a 10V battery, the current through the resistor is measured to be 0.5A,

(a) What is the resistance of the resistor?

(b) What is the resistivity of the resistor?

(c) What electric field is produced within the resistor?

(d) How much heat is the resistor producing each second?

(a) SINCE THE RESISTOR IS ALONE CONNECTED TO THE BATTERY, THEY ARE IN PARALLEL ⇒ THE VOLTAGE OF THE RESISTOR IS 10V. USING OHM'S LAW, WE CAN FIND THE RESISTANCE:

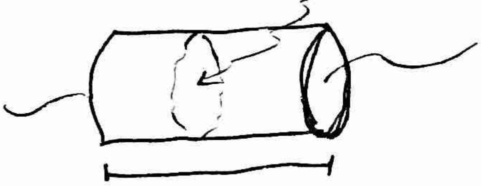
$$V = iR \Rightarrow R = \frac{V}{i} = \frac{(10)}{(0.5)} = \boxed{20\,\Omega}$$

(WE WERE TOLD CURRENT = 0.5A)

(b) FOR A CYLINDRICAL RESISTOR:

$$A = \pi r^2 = \pi(0.001)^2 = 3.14 \times 10^{-6}\,m^2$$

1mm = 0.001m



L = 5cm = 0.5m

$$R = \rho\frac{L}{A} \Rightarrow \rho = \frac{RA}{L} = \frac{(20)(3.14\times10^{-6})}{(0.5)}$$

$$= \boxed{1.26 \times 10^{-4}\,\Omega m}$$

(c) $$E = \frac{V}{L} = \frac{(10)}{(0.05)} = \boxed{200\ N/C}$$

(5cm = 0.05m)

(d) POWER = ENERGY/TIME, & HEAT IS THE ENERGY EMITTED BY A RESISTOR.

$$\Rightarrow P = Vi = (10)(0.5) = 5W = \boxed{5\text{ J OF HEAT PER SECOND}}$$

FORMULA SHEET

- Vectors:

$$\vec{A} \cdot \vec{B} = AB\cos\theta = A_x B_x + A_y B_y + A_z B_z$$

$$\left|\vec{A} \times \vec{B}\right| = AB\sin\theta$$

- Physics I Formulae:

$$\sum \vec{F} = m\vec{a}$$

$$W = \vec{F} \cdot \Delta\vec{x}$$

$$W_{tot} = \Delta K$$

$$W_{cons} = -\Delta U$$

$$K = \frac{1}{2}mv^2$$

$$K_i + U_i = K_f + U_f$$

- Electric Forces:

$$e = 1.6 \times 10^{-19}\ \mathrm{C}$$

$$\left.\begin{array}{l} k = 8.99 \times 10^{9}\ \dfrac{\mathrm{Nm^2}}{\mathrm{C^2}} \\ \epsilon_0 = 8.85 \times 10^{-12}\ \dfrac{\mathrm{F}}{\mathrm{m}} \end{array}\right\} \quad k = \frac{1}{4\pi\epsilon_0}$$

$$Q = Ne$$

$$F = k\frac{q_1 q_2}{r^2}$$

$$\vec{F} = q\vec{E}$$

- Electric Fields:

$$E = k\frac{q}{r^2} \quad \text{(point charge)}$$

$$E = \frac{\lambda}{2\pi\epsilon_0} \quad \text{(infinite line of charge)}$$

$$E = \frac{\sigma}{2\epsilon_0} \quad \text{(infinite sheet of charge)}$$

$$\Phi_E = \vec{E} \cdot \vec{A}$$

$$\Phi_{tot} = \frac{q_{enc}}{\epsilon_0}$$

$$\lambda = \frac{Q}{L} \quad \text{or} \quad \sigma = \frac{Q}{A} \quad \text{or} \quad \rho = \frac{Q}{V} \quad \text{(charge densities)}$$

- Electric Potential Energy and Electric Potential:

$$U = k\frac{q_1 q_2}{r}$$

$$\phi = k\frac{q}{r}$$

$$U = q\phi \quad \text{and} \quad \Delta U = q\Delta\phi$$

$$E_{av} = \frac{\Delta\phi}{\Delta x}$$

$$V = \Delta\phi$$

- Capacitors:

$$Q = CV$$

$$C = \epsilon_0\frac{A}{d}$$

$$E = \frac{V}{d}$$

$$U = \frac{1}{2}\frac{Q^2}{C}$$

$$u = \frac{1}{2}\epsilon_0 E^2$$

- Resistors:

$$R = \rho\frac{L}{A}$$

$$V = iR$$

$$E = \frac{V}{L}$$

$$P = Vi$$